Using complete sentences and proper mathematical notation, write the formal definition of "continuous (at a point)". **SCORE: _____ / 2 PTS**

f IS CONTINUOUS AT a IFF f(a) EXISTS,

GRADED BY ME

$\lim_{x \to a} f(x)$ EXISTS AND

$\lim_{x \to a} f(x) = f(a)$

Using complete sentences and proper mathematical notation, write the formal definition of "derivative (function)". **SCORE: _____ / 1 PT**

THE DERIVATIVE OF f IS $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ GRADED BY ME

The time it takes to recover from a certain illness depends on the daily dosage of a certain medicine. **SCORE: _____ / 2 PTS**
Suppose $r = f(d)$, where r is the recovery time (in days), and d is the daily dosage (in grams)

[a] What does $f(10) = 6$ mean ? Give the correct units for all numbers in your answer.

IT TAKES 6 DAYS TO RECOVER IF THE DAILY DOSAGE IS 10g.

[b] What does $f'(10) = -0.5$ mean ? Give the correct units for all numbers in your answer. GRADED BY ME

IF THE DAILY DOSAGE IS 10g,
YOU WILL RECOVER $\approx \frac{1}{2}$ DAY SOONER FOR EACH ADDITIONAL 1g YOU TAKE EACH DAY

Find the following limits. **SCORE: _____ / 7 PTS**
Each answer should be a number, ∞, $-\infty$, or DNE (only if the other answers do not apply).

[a] $\lim_{x \to -\infty} \tan^{-1} x$

$= \boxed{-\frac{\pi}{2}}$ 1

[b] $\lim_{x \to \infty} \arccos e^{-x}$

$= \boxed{\arccos 0}$ 1

$= \boxed{\frac{\pi}{2}}$ 1

[c] $\lim_{x \to -\infty} \frac{\sqrt{49x^2 - 36x}}{5 - 4x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x \to -\infty} \frac{\sqrt{49x^2 - 36x} \cdot -\sqrt{\frac{1}{x^2}}}{\frac{5}{x} - 4} = \lim_{x \to -\infty} \boxed{\frac{-\sqrt{49 - \frac{36}{x}}}{\frac{5}{x} - 4}}$ 1 2 $= \frac{-\sqrt{49 - 0}}{0 - 4} = \boxed{\frac{7}{4}}$ 1

Prove that $\tan x = \cos x$ for some x in the interval $(0, \frac{\pi}{4})$. **DO NOT ATTEMPT TO SOLVE FOR x.** **SCORE: _____ / 4 PTS**

LET $f(x) = \tan x - \cos x$. f IS CONTINUOUS ON $[0, \frac{\pi}{4}]$ SINCE IT IS THE DIFFERENCE OF CONTINUOUS FUNCTIONS

1

$f(0) = \tan 0 - \cos 0 = -1$ AND $f(\frac{\pi}{4}) = \tan\frac{\pi}{4} - \cos\frac{\pi}{4} = 1 - \frac{\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$

$-1 < 0 < \frac{2-\sqrt{2}}{2}$, SO BY IVT, FOR SOME $c \in (0, \frac{\pi}{4})$, $f(c) = 0$

IE. $\tan c - \cos c = 0$ OR $\tan c = \cos c$

1 1 1

Let $f(x) = \sqrt{29-4x}$. **SCORE: _____ / 8 PTS**

[a] Find $f'(x)$.

1 $\lim_{h\to 0} \frac{\sqrt{29-4(x+h)} - \sqrt{29-4x}}{h} \cdot \frac{\sqrt{29-4(x+h)} + \sqrt{29-4x}}{\sqrt{29-4(x+h)} + \sqrt{29-4x}}$

$= \lim_{h\to 0} \frac{29-4(x+h) - (29-4x)}{h(\sqrt{29-4(x+h)} + \sqrt{29-4x})}$

2 $= \lim_{h\to 0} \frac{-4\cancel{h}}{\cancel{h}(\sqrt{29-4(x+h)} + \sqrt{29-4x})} = \frac{-4}{2\sqrt{29-4x}} = \frac{-2}{\sqrt{29-4x}}$ 1

[b] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x = 1$.

$f'(1) = \frac{-2}{\sqrt{25}} = -\frac{2}{5}$ 1 $y - 5 = -\frac{2}{5}(x-1)$ 1

[c] The position (in yards) of an object moving in a straight line is given by $s(t) = \sqrt{29-4t}$, where t is the time in minutes. Find the instantaneous velocity of the object at time $t = 5$. Give the correct units for your answer.

$s'(5) = \frac{-2}{\sqrt{9}} = -\frac{2}{3}$ 1 YARD / MINUTE 1

Determine if each of the following functions is continuous. **STATE YOUR CONCLUSIONS CLEARLY.** **SCORE: _____ / 6 PTS**
If a function is continuous, justify your conclusion using the definition(s) and/or theorems.
If a function is not continuous, show clearly which part of the definition of "continuous" is not true.

[a] $f(x) = \begin{cases} \frac{x^3+1}{x^2-1}, & \text{if } x < -1 \\ \frac{x^2-4}{x+3}, & \text{if } x > -1 \end{cases}$

$f(-1)$ DNE
f NOT CONT
1

[b] $f(x) = \begin{cases} x^4 - x^3 - 1, & \text{if } x \le 2 \\ x^5 - 10x - 5, & \text{if } x > 2 \end{cases}$

1
f IS CONT AT $x \neq 2$ (POLYNOMIAL)

$f(2) = 2^4 - 2^3 - 1 = 7$

½ $\lim_{x\to 2^+} (x^5 - 10x - 5) = 2^5 - 10(2) - 5 = 7$

½ $\lim_{x\to 2^-} (x^4 - x^3 - 1) = 2^4 - 2^3 - 1 = 7$

1 $\lim_{x\to 2} f(x) = 7 = f(2)$ 1

f IS CONT. AT $x = 2$

SO f IS CONT. 1